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Structural analysis of a chair side frame connected with semi-rigid joints by slope-deflection method

Ersan Güray[1✉], Ali Kasal[2], Şeyma Aktürk[3], Tolga Kuşkun[2], Jerzy Smardzewski[4]

[1]Muğla Sıtkı Koçman University, Faculty of Engineering, Department of Civil Engineering, Muğla, Turkey
[2]Muğla Sıtkı Koçman University, Faculty of Technology, Department of Wood Science and Industrial Engineering, Muğla, Turkey
[3]Doğtaş Kelebek Mobilya A.Ş. Bandırma Yolu 5. Km Biga, Çanakkale, Türkiye
[4]Poznan Universtiy of Life Science, Faculty of Forestry and Wood Technology, Department of Furniture Design, Poznań, Poland

Key words

furniture
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slope-deflection method
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semi-rigid joints

Abstract

Frame furniture structures are formed by beams and connections behave as semi-rigid joints. Fur-niture engineering process that including the structural analyses should be developed to produce suitable furniture designs in the intersection of art and science. Due to excessive lateral displace-ments, connection points become highly critical to face with a sudden failure. At that point, slope deflection method can be used as a valuable method for calculating the critical moment values at the member ends that will occur as a result of relatively large lateral displacements, especially in the case of lateral loads, for the flexible furniture with semi-rigid connections. Although the slope deflection method is a fundamental, classical method for analysis, specifically, slope-deflection method was never been utilized for analysis of wooden furniture frame structures. In this study, the slope-deflection method was utilized to a representative side frame of chair, which also in-cludes semi-rigid connections in the problem, to facilitate the static analysis. By reducing and in-creasing the stiffness/rigidity of joints, the displacements were evaluated and it was shown that this method could also be used efficiently in structural analysis of furniture frames.

✉Ersan Güray, Department of Civil Engineering, Faculty of Engineering, Muğla Sıtkı Koçman University, Muğla, Turkey, e-mail: ersanguray@mu.edu.tr

Introduction

Although furniture is a structural system, it has not been the subject of mathe-matical theories until recently, and science of furniture engineering has still not been systematically applied in the Furniture Industry (Kasal, 2004). However, it is becoming increas-ingly important because of the various reasons such as; consumer demands for more reliable furniture, government pressures for warranties of furniture, increasing needs for material economics, etc. According to Eckelman furniture is the forgotten child of engineering sciences (Eckelman, 2003). Furniture engineering should be developed to produce suita-ble furniture designs in the intersection of art and science. So, this forgotten child should be raised and related studies should be carried out to contribute to furniture engineering field.

In the engineering design of the furniture, firstly, the forces acting on the ends of members have been analyzed by means of various structural analysis methods, then, stresses in these members could be calculated and by comparing with the acceptable design stresses for the materials used, thus, it can be seen whether or not each of the members has been safely designed. This process provides a methodological approach for designing a piece of furniture to meet any specified service condition.

In general for furniture products, three essential construction methods were used. Furniture was built by the case or frame method or by a combination of the two, the complex method. The case method means that the elements of furniture were plates. In the frame method, the elements of furniture were beams instead of panels (Eckelman, 2003). Frame structures can be considered as structures where beams are connected each other to form a skeleton. Beams, beam columns, slabs inside resist against lateral and gravity loads. Similar in the case of the civil engineering applications such like buildings etc., in furniture industry, specifically in the case of simple chair furniture, the legs, the side-rails and the lateral member at the sides of the sitting part forms a frame structure.

Structural analysis methods have been developed in almost last two decades. Just after 1850's, there had been published very beginning of the researches about practical methods for structural analysis and flexure of continuous beams (Rojas, 2012). Maxwell and Castigliano made great contributions to the development structural analysis by using Work and Energy principles in late 19th century. Hardy-Cross who was the Professor at the University of Illinois developed moment distribution method depending on successive steps to determine end moments of segments of a continuous beam (Cross, 1932). Although it was a great method for its age, its applications had some limitations. First, the method does not include deflections caused by axial strains and secondly, and the most important limitation is that the entire lateral movement, side-sway is excluded (Leach, 1960). Slope deflection method is introduced as a publication at 1915 by Maney (Wilson and Maney, 1915). In the furthering years, slope deflection method became a key point to derive the stiffness method which is formulated to be utilized practically by means of matrix systems (Hibbeler, 2008). In the 1950s, developments in structural analysis methods went a step further and extended to finite element analysis.

At the beginning, slope deflection method is applied to the frame structures at which all members are assumed to be Euler- Benoulli beams such that all deformations, e.g. slopes and deflections are assumed to be reasoned by the internal moments such that the deformations by axial or shear forces are neglected. A set of equations is derived by means of the moment area theorems, a contribution by Otto Mohr, and the conjugate beam method, derived by H. Müller-Breslau (Samuelsson and Zienkiewicz, 2006). Moment area theorems state, the area under the curve of the moment diagram between any two nodes related linearly to the change in slope between these points and the flexural rigidity of the member. Equation for each member ending is derived by conjugate beam method such that the moment diagram of the actual case is used as the loading configuration and corresponding shear and moment diagrams refer to slope and deflection through the beam, respectively. Once all equations are solved simultaneously, the internal moments are obtained then all slopes (rotations), θ and deflections (displacements), Ψ are obtained at each node. These equations were enriched by the addition of axial unit deformations and presented as a matrix and computer program to be used in the analysis of multi-storey structures fixed with shear walls (Goldberg, 1967). This method has been used not only to analyze frame structures. Backer et. al. demonstrated an analytical method with slope-deflection formulas for bridge deck analysis. Internal moments in the trapezoidal stiffeners forming the orthotropic bridge deck are determined by this analytical approach (Backer et al., 2008). In another study, slope deflection formulations are derived for the tapered beam-columns to determine the critical buckling loads (Riahi and Etezady, 2012). J.D.A. Ochoa developed 2nd order slope-deflection formulations by including the combined effects of shear and bending deformations and 2nd order axial loading effects on Timoshenko beam supported with semi-rigid connections (Ochoa, 2008; Ochoa, 2010).

The exact analysis of furniture frames is a computationally complex process ow-ing to the large number of internal forces which must be determined in order to make an effective solution. It is very important to use of technological possibilities in strength design of furniture. Today, engineering design of furniture can be accom-plished by utilizing solid modeling and structural analysis softwares. From a practic-ing furniture engineer's point of view, finite element method (FEM) provide the most convenient tool for analyzing the furniture systems. Furniture members, joints, and en-tire of the system can be modelled parametrically via FEM. Then, the strength calcula-tions of the designed members, joints, and the whole system could be made by simu-lating real conditions and loads to solid modelled system, so the stresses at node or member can be obtained.

Recent studies show that the FEM, in the structural analysis of furniture systems and joints has become widespread. A review of the FEM applied in the analysis of furniture constructed with wood materials was given in the study (Tankut and Tankut, 2014).

In the study (Gustafsson, 1997), it was shown how to analyze and design a chair with FEM, and gave the experimental results with stress-strain diagrams of the real chair constructed of Swedish ash (Gustafsson, 1997), also stated that analyzing furniture is a difficult and time-consuming work such that this problem can be solved with FEM software. Kasal, et. al. created analytical models for structural analyses of furniture frames by FEM. It was concluded that analytical models created by FEM give the information concerning the deformations and internal forces on furniture members and joints (Kasal and Puella, 1995). Results of direct stiffness method and FEM for a chair model were compared (Hajdarevic and Busuladzic, 2015). Numerical calculations were performed with the assumption of linear elastic model for orthotropic materials. While both methods gave very good results, it was seen that the direct stiffness method gave more reliable results, especially when semi-solid connections were defined. In another study; the principles of furniture design and to maximize the strength of members and joints while minimizing material use was aimed to determine. For this purpose, a chair side frame was analyzed with FEM software and proved that this developed program can analyze the rigidity and strength of wooden furniture accurately and quickly (Smardzewski, 1998). Laminated wood-based seating furniture was modelled by FEM and stress analyzes and deformations to provide necessary servicability were dissussed (Langová et al., 2019). Size effects on determining the mechanical properties of wooden specimen was emphasized (Hu, et al., 2019), by comparisons of results from experimental and finite element models. The research regarding the numerical and analytical analyses of the withdrawal strength of the T-type mortise and tenon joints was carried out by (Hu and Guan, 2017). In conclusion, it was demonstrated that the consistency level between the withdrawal test results and numerical method was 83%, while it was 80% between the test results and analytical method (Hu and Guan, 2017). A novel joint type is modelled and analyzed (Chen et al., 2022). The stress concentrations on joint reinforcement are well estimated by its finite element model. Ceylan et. al. (2021) analyzed the chair side frame both numerically and experimentally. Radial spring coefficients of individual tenon-mortise joints were determined, then inserted in numerical calculations such that numerical results and experiments fitted well. The other research was carried out to develop a new model of mortise and tenon joint considering the glue line and friction coefficient to analyze the semi-rigid wood joints. The results of the study showed that the developed model of the joint could be applied to analyze the semi-rigid mortise and tenon joint by 85% consistency level (Hu and Guan, 2019). In the other study, three different FEM models used to predict the bending strength of mortise and tenon joints were compared. The results indicated that the semi-rigid model performed much better than the tie rigid model, followed by the whole rigid model (Hu and Liu, 2020). In the study (Hitka et al., 2018), load-carrying capacity and the size of chair joints were studied numerically for chairs on a FEM based software. Legs were constructed by joints where they where attached either with rails or supplemented with braces, or with rails and side stretchers. As it can be understood from the literature, the reliability of the results obtained from the structural analyses is directly related to the correct definition of the materials used in the structural system, and especially the joints into the structural analysis program. In particular, the accuracy in defining the joints affects the force and moment distributions and displacements that will occur in the joints under loading conditions.

Understanding the structural characteristics of furniture depends on the mechanical behavior of the joints used in its construction. Furniture joint can be defined as a certain system that is formed as a result of connecting the ends of the members with appropriate tools and techniques. Furniture joints are semi-rigid connections, and the semi-rigidity coefficient values for each joint type should be determined by experiments.

The behavior of the joints is generally not taken into account in the current structural analysis applied in fields such as civil engineering, especially in steel structures. For this reason, there are two types of connection assumptions at the connections, fully rigid (fixed) or flexible (pinned). It is assumed that all the moment is transferred without any rotation in rigid connections, while in flexible connections there is freedom of rotation and no moment is transferred. However, in practice, a third type of semi-rigid connection, which both makes some rotation and transfers some moment, is encountered. Hence, the concept of a third connection classification, called semi-rigid joint, has emerged (Fig. 1a, b).

In the analysis of large structures as buildings, etc., the connection points constitute a very small place in the structure such that the joint behaves almost fixed.

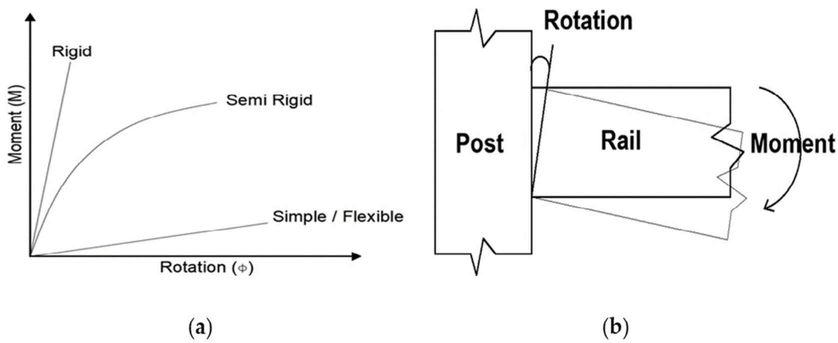

Fig. 1. Rigid, semi-rigid and flexible connections in the moment-rotation relationship (a) and mechanical behavior of semi-rigid connection in the post to rail joint (b)

Contrarily, the joints constitute a larger place in smaller structural systems such as chairs, sofa frames and tables. Therefore, in the structural analysis of furniture, the effects of the mechanical behavior of the joints on the analysis results will be greater. For this reason, it is necessary to define the joints as semi-rigid in the structural analysis of furniture systems.

In this study, slope deflection method is applied to the representative chair side frame for the structural analysis. The chair frame has semi-rigid connections as in the practical applications. Apart from the civil engineering applications, in the case of a furniture frame subjected to sidethrust loads, the joints are exposed to serious angular deformations and lateral displacements (sway), such that the joints cannot keep their rotational rigidity after a limit. Consequently, semi-rigid modelling of connection points would be highly compatible to the case.

The methodology of this paper could be expressed in following sections. Slope deflection equation for a simple beam was derived at section 2. The geometry of the problem, the set of equations were derived for the chair model and results were introduced and discussed, then tables, figures and other outcomes were given in section 3. Conclusions are presented briefly in section 4.

Methods and materials

Angular deformations and lateral displacements are formulated by conjugate beam method. As given in Fig.2a and 2b, moments, M'AB and M' BA are applied to the ends of the beam, independently, resulting slopes or angular displacements at end A and end B, are derived as (Hibbeler, 2008):

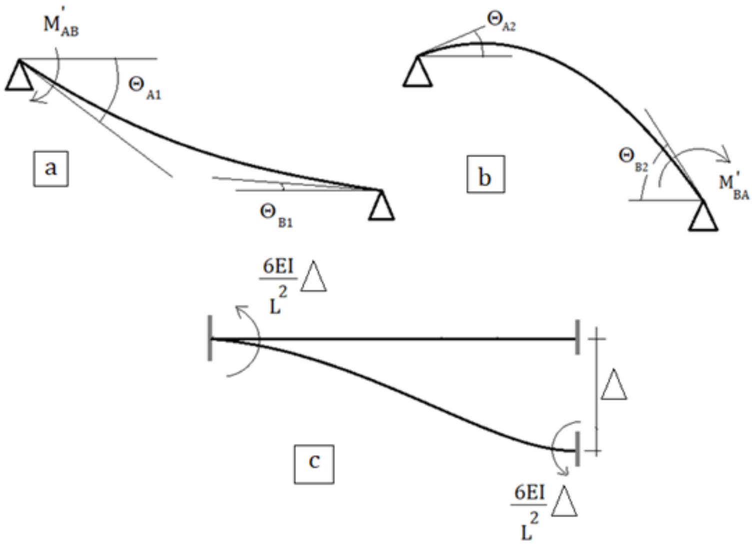

Fig. 2. End rotation at A (a) End rotation at B (b) and lateral displacement at B (c) with corresponding end moments

$$\theta_{A1} = (M'_{AB}L)/3EI \quad (1)$$

$$\theta_{B1} = (M'_{AB}L)/6EI \quad (2)$$

$$\theta_{A2} = (M'_{BA}L)/6EI \quad (3)$$

$$\theta_{B2} = (M'_{BA}L)/3EI \quad (4)$$

These independent angular displacements are superposed such that the angular deformations at both ends are calculated as given:

$$\theta_A = \theta_{A1} - \theta_{A2} \quad (5)$$

$$\theta_B = \theta_{B1} - \theta_{B2} \quad (6)$$

Inserting θ_{A1}, θ_{B1}, θ_{A2} and θ_{B2} into eqn. 5 and 6 will result in a set of equations. The simultaneous solution of them ends up with the end moment equations as given below:

$$M'_{AB} = \frac{2EI}{L}\left(2\theta_A + \theta_B\right) \quad (7)$$

$$M'_{BA} = \frac{2EI}{L}\left(\theta_A + 2\theta_B\right) \quad (8)$$

Finally, end moments, M_{AB} and M_{BA}, are obtained by addition of angular displacement and lateral displacement contributions. Here Δ refers to the lateral displacement (sway) and FEM refers to fixed end moments due to loadings located not exactly to the nodes but between nodes. In this study, load is a point load and applied to a node exactly. End moments are:

$$M_{AB} = M'_{AB} - \frac{6EI}{L^2}\Delta + FEM_A \tag{9}$$

$$M_{BA} = M'_{BA} - \frac{6EI}{L^2}\Delta + FEM_B \tag{10}$$

Results and discussion

Side of the chair is supposed to be a frame structure. AE, FI, BH, CG and DF members form the skeleton (Fig. 3a and 3b). BH, CG and DF members are called side-rails and connected to single piece members, AE and FI by semi-rigid connections.

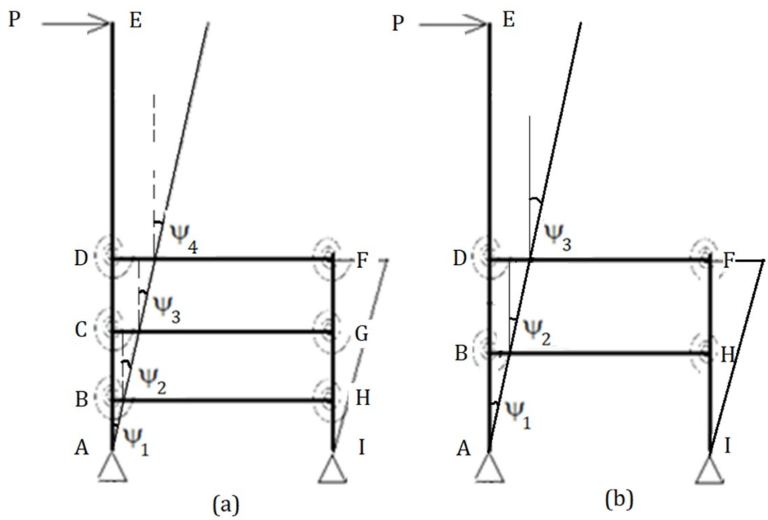

Fig. 3. Model of the chair side frame with a) double side- rails b) single side-rail

It is supported by pin connections at A and I nodes. Semi-rigid connections are modelled by using radial springs, with spring constant k (N-cm/rad), at any arbitrary semi-rigid connection (Fig. 4), the displacement of the node on the vertical member

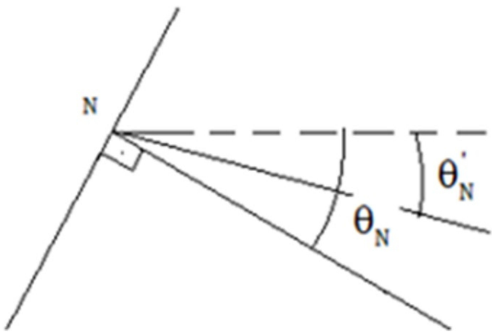

Fig. 4. Angular displacements at an arbitrary node N

(θN) and the displacement of the node on the lateral member (θ›N) are intro-duced sep-arately.

Semi-rigidity is obtained by means of radial linear springs. It is compressed or tensioned at an amount of the difference of these angular dis-placements. Since the frame subject-ed to lateral loading, sway occurs. It is defined by means of individual sway angles Ψ1, Ψ2, Ψ3 and Ψ4,

The angular displacements at each node and sway angles are calculated by the slope deflection equations, the moment equilibrium equations at nodes and the equi-librium of lateral loads. As a result of the force applied from point E, sway will occur in the frame. At the same time, angular displacements will occur at all nodes. Since A and I points are pin connections, there is no need to write a moment formulation here. The moment equilibrium equations for remaining nodes, B, C, D, F, G, and H, can be writ-ten as:

$$M_{BA} + M_{BC} + M_{BH} = 0 \tag{11}$$

$$M_{CB} + M_{CG} + M_{CD} = 0 \tag{12}$$

$$M_{DC} + M_{DF} + M_{DE} = 0 \tag{13}$$

$$M_{HG} + M_{HB} + M_{HI} = 0 \tag{14}$$

$$M_{GF} + M_{GC} + M_{GH} = 0 \tag{15}$$

$$M_{FD} + M_{FG} = 0 \tag{16}$$

Slope deflection equations are constructed for the prescribed moments as shown at eqn.'s 7, 8, 9 and 10. Slope deflection equations are derived for the beam which has fixed ends at both ends. Here, AB and IH members have pin ends at supports and DE has a free end at E, so that MAB, MIH and MED are not calculated and MBA, MHI and MDE are rearranged such that:

$$M_{BA} = 3\frac{E_{BA}I_{BA}}{L_{BA}}\left(\theta_B - \Psi_1\right) \tag{17}$$

$$M_{BC} = 2\frac{E_{BC}I_{BC}}{L_{BC}}\left(2\theta_B + \theta_C - 3\Psi_2\right) \tag{18}$$

$$M_{BH} = 2\frac{E_{BH}I_{BH}}{L_{BH}}\left(2\theta'_B + \theta'_H\right) \tag{19}$$

$$M_{CB} = 2\frac{E_{CB}I_{CB}}{L_{CB}}\left(2\theta_C + \theta_B - 3\Psi_2\right) \tag{20}$$

$$M_{CD} = 2\frac{E_{CD}I_{CD}}{L_{CD}}\left(2\theta_C + \theta_D - 3\Psi_3\right) \tag{21}$$

$$M_{CG} = 2\frac{E_{CG}I_{CG}}{L_{CG}}\left(2\theta'_C + \theta'_G\right) \tag{22}$$

$$M_{DC} = 2\frac{E_{DC}I_{DC}}{L_{DC}}\left(2\theta_D + \theta_C - 3\Psi_3\right) \tag{23}$$

$$M_{DE} = 3\frac{E_{DE}I_{DE}}{L_{DE}}\left(\theta_D + \Psi_4\right) \tag{24}$$

$$M_{DF} = 2\frac{E_{DF}I_{DF}}{L_{DF}}\left(2\theta'_D + \theta'_F\right) \tag{25}$$

$$M_{HI} = 3\frac{E_{HI}I_{HI}}{L_{HI}}\left(\theta_H + \Psi_1\right) \tag{26}$$

$$M_{HG} = 2\frac{E_{HG}I_{HG}}{L_{HG}}\left(2\theta_H + \theta_G + 3\Psi_2\right) \tag{27}$$

$$M_{HB} = 2\frac{E_{HB}I_{HB}}{L_{HB}}\left(2\theta'_H + \theta'_B\right) \tag{28}$$

$$M_{GH} = 2\frac{E_{GH}I_{GH}}{L_{GH}}\left(2\theta_G + \theta_H - 3\Psi_2\right) \tag{29}$$

$$M_{GF} = 2\frac{E_{GF}I_{GF}}{L_{GF}}\left(2\theta_G + \theta_F - 3\Psi_3\right) \tag{30}$$

$$M_{GC} = 2\frac{E_{GC}I_{GC}}{L_{GC}}\left(2\theta'_G + \theta'_C\right) \tag{31}$$

$$M_{FG} = 2\frac{E_{FG}I_{FG}}{L_{FG}}\left(2\theta_F + \theta_G - 3\Psi_3\right) \tag{32}$$

$$M_{FD} = 2\frac{E_{FD}I_{FD}}{L_{FD}}\left(2\theta'_F + \theta'_D\right) \tag{33}$$

Here E is the elasticity constant, I is the moment of inertia of the section, and Ψ is the angular change due to translation. The moment of the semi-rigid connected elements at the nodes can be expressed as follows, in accordance with the linear spring model:

$$M_{BH} = k_B(\theta_B - \theta'_B) \quad (34)$$

$$M_{HB} = k_H(\theta_H - \theta'_H) \quad (35)$$

$$M_{CG} = k_C(\theta_C - \theta'_C) \quad (36)$$

$$M_{GC} = k_G(\theta_G - \theta'_G) \quad (37)$$

$$M_{DF} = k_D(\theta_D - \theta'_D) \quad (38)$$

$$M_{FD} = k_F(\theta_F - \theta'_F) \quad (39)$$

The k constants in the above equations are the spring constant values of the spiral spring used in the model of the semi-rigid element connected at the relevant node.

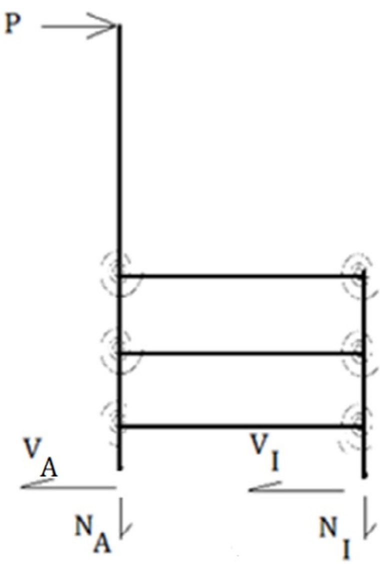

Fig. 5. Free body diagram of the chair side frame with double side-rails

Last three equations can be written according to the shear forces at the vertical members of the system (Fig. 5). Here, since the sum of shear on AB and HI will be equal to the horizontally applied load:

$$P - V_A - V_I = P + \frac{M_{BA}}{L_{BA}} + \frac{M_{HI}}{L_{HI}} = 0 \quad (40)$$

Similar to this, for 2nd and 3rd stages, equations can be derived as:

$$P + \frac{(M_{BC} + M_{CB})}{L_{BC}} + \frac{(M_{GH} + M_{HG})}{L_{GH}} = 0 \quad (41)$$

$$P + \frac{(M_{CD} + M_{DC})}{L_{CD}} + \frac{(M_{GF} + M_{FG})}{L_{GF}} = 0 \quad (42)$$

Eqn.'s 11–42 are solved simultaneously to determine angular deformations, lateral displacements and end moments.

By taking the simpler structure, to focus on the results and to show that the simple problem can be solved with this method, the problem is solved with a single single sid-erail such that the BH element is preserved, while the CG element is removed (Fig. 3b). Here, all elements are identical in cross-sectional sizes and the moments of inertia of the elements are calculated for 60 mm in depth and the 21 mm in width of section di-mensions. The modulus of elasticity is taken as 10 GPa. to illustrate a wooden material. The highest point of the frame is 90 cm, the seat height is 41 cm, the seat depth is 45 cm. and the side-rail is at a height of 21 cm, consequently, AB and HI members are 21 cm, BD and HF are 20 cm, BH and DF are 45 cm and the DE member is 49 cm in length. The force P is taken to be 1000 N. B, D, F and H joints are supposed to be semi-rigid connections such that all these semi-rigid connections linear radial spring coeffients are inserted as kB =kD = kF = kH = C ·105 N-cm/rad where C is an integer from [1, 2, 5, 12, 120, 200, 1000] set. It extends from 1 to 1000 to represent the spring rigidity then high-er values models very rigid connections.

As the C number increases, slopes of lateral members and slopes of vertical members at any same joint converge (Table 1). In the case of very rigid connections, they get closer and closer, e.g. when k is 108 Ncm/rad (C=1000), their ratio approximates to a percentage value between 92 to 96 (Table 2). Sway angles Ψ1, Ψ2 and Ψ3 refers to the these angles at joints B, D and E respectively. They decrease by increasing the stiffness of the semi-rigid connections. While this reduce seems more apparent for the nodes A and B, the sway angle at joint D does not drop as much as at A and B

Table 1. Slopes of vertical members (Θ) and lateral members (Θ') at nodes B, H, D and F for various C semi-rigid connection constants

Slopes (rad)	C						
	1	2	5	12	120	200	1000
Θ_B	0.2266	0.1141	0.0468	0.0208	0.0046	0.0039	0.0031
Θ'_B	0.0044	0.0043	0.0041	0.0038	0.0031	0.0030	0.0029
Θ_H	0.2287	0.1163	0.0488	0.0227	0.0062	0.0055	0.0046
Θ'_H	0.0045	0.0045	0.0045	0.0046	0.0045	0.0045	0.0044
Θ_D	0.2350	0.1224	0.0547	0.0281	0.0103	0.0095	0.0085
Θ'_D	0.0047	0.0049	0.0054	0.0062	0.0078	0.0080	0.0082
Θ_F	0.2275	0.1150	0.0476	0.0213	0.0042	0.0035	0.0025
Θ'_F	0.0043	0.0041	0.0038	0.0033	0.0025	0.0024	0.0023

Table 2. Ratio of slopes of lateral members (Θ') to the corresponding slopes of vertical members (Θ) at nodes B, H, D and F for various C semi-rigid connection constants

Ratio (%)	C						
	1	2	5	12	120	200	1000
Θ'_B/Θ_B	1.9	3.8	8.7	18.2	67.4	77.4	94.4
Θ'_H/Θ_H	2.0	3.9	9.3	20.2	72.7	81.6	95.7
Θ'_D/Θ_D	2.0	4.0	9.9	21.9	75.5	83.8	96.3
Θ'_F/Θ_F	1.9	3.6	8.0	15.7	58.0	69.1	91.5

(Table 3). As a result of this, deflections at the B, D and E joints also decreases due to the increase in stiffness (Table 4).

Table 3. Sway angles (Ψ) at nodes A, B and D for various C semi-rigid connection constants

Sway angles (rad)	C						
	1	2	5	12	120	200	1000
Ψ_1	0.2296	0.1171	0.0498	0.0237	0.0073	0.0066	0.0058
Ψ_2	0.2299	0.1174	0.0499	0.0237	0.0068	0.0060	0.0051
Ψ_3	0.2562	0.1436	0.0759	0.0493	0.0315	0.0307	0.0297

Table 4. Deflections at nodes B, D and E for various C semi-rigid connection constants

Deflections (cm)	C						
	1	2	5	12	120	200	1000
Δ_B	4.82	2.46	1.04	0.50	0.15	0.14	0.12
Δ_D	9.42	4.81	2.04	0.97	0.29	0.26	0.22
Δ_E	21.97	11.84	5.76	3.38	1.83	1.76	1.68

To investigate the relationship between the connection rigidity and the displacements at member ends B, D and E, best fit lines are determined (Fig. 6). These lines show the different trends of deflections at member ends B, D and E for the stiffness

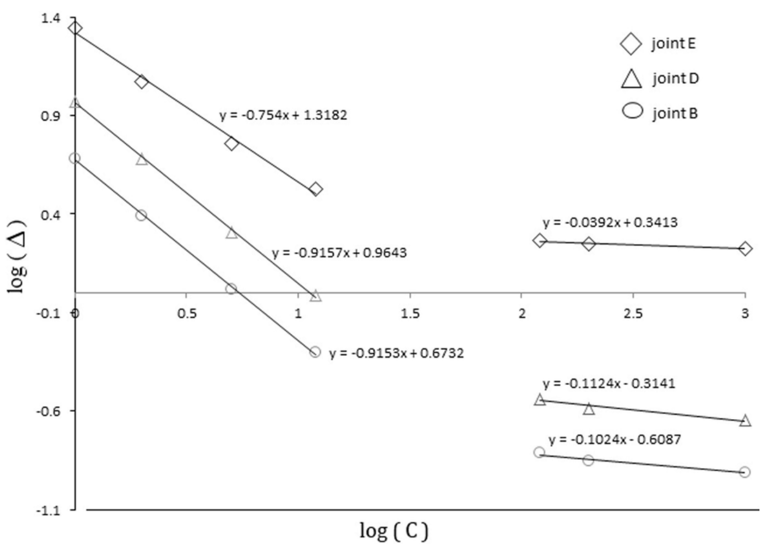

Fig. 6. Semi-rigid joints stiffness vs. the lateral deflection at joints E, D and B in logarithmic scale and the best fit lines

of the connection points before and after some certain values. These values can be obtained by intersection points of referring lines and determined C values are 37.8, 39.0 and 23.3 for B, D and E joints, respectively. Beside that, it is observed from the lines, slope of the line for E is highly less than the slopes of lines B and D such that the deflection at joint E does not decrease as much as at other points (B and D) with the increase of the spring constant.

In Table 5, the end moments are tabulated for C values 1, 2, 5, 12, 120, 200 and 1000. While the rigidity of the semi-rigid connection increases, end moments decrease at all members except for DF member. As the system becomes more rigid with the in-crease in the rigidity at the connection points, all the elements carry less moment, while the resistance against to the horizontal load is provided by DF member by car-rying more and more internal moment. In fact, when the C value is 1000, end moment carried by the DF member is 36 percent more when compared with the case when the C value is 1. It should be noticed that end moment at DE member is 49 kN-cm as usual for all cases.

Table 5. Moments (kN-cm) at member ends for various C semi-rigid connection constants

C	M_{BA}	M_{BH}	M_{BD}	M_{DB}	M_{DF}	M_{DE}	M_{FH}	M_{FD}	M_{HI}	M_{HB}	M_{HF}
1	–16.34	22.22	–5.88	25.97	23.03	–49.00	–22.32	22.32	–4.66	22.43	–17.76
2	–16.22	21.97	–5.75	25.50	23.50	–49.00	–22.18	22.18	–4.78	22.35	–17.57
5	–15.95	21.36	–5.41	24.37	24.63	–49.00	–21.88	21.88	–5.05	22.14	–17.09
12	–15.57	20.41	–4.84	22.69	26.31	–49.00	–21.55	21.55	–5.43	21.73	–16.30
120	–14.81	17.93	–3.13	18.62	30.38	–49.00	–21.40	21.40	–6.19	20.29	–14.09
200	–14.74	17.67	–2.93	18.22	30.78	–49.00	–21.44	21.44	–6.26	20.11	–13.85
1000	–14.66	17.31	–2.66	17.67	31.33	–49.00	–21.50	21.50	–6.34	19.86	–13.51

Conclusions

In this study, a representative chair side frame was structurally analyzed by using slope-deflection method. Connections in the system were assumed to be semi-rigid in the structural analyses. By reducing and increasing the stiffness/rigidity of joints, the displacements were evaluated.

Furniture frames are exposed large lateral deflections (sway) in practical use due to semi-rigid connections in general. The analysis of such frames would require in-cluding not only the rotations at joints but also the lateral displacements in the present case. In Tables 1–4, it is shown that, reasonable amount of lateral displacements are ob-tained when the representative chair frame is subjected to almost twice of house-use load and increasing rigidity in the joints causes less sway. In addition to this, as shown in Table-5, by increasing stiffness at connections, end moments reduces slightly at all members except for DF which is the top side lateral member that is subjected to more bending moment. In fact, when the most flexible and rigid cases those when C is 1 and 1000, respectively, are compared, the lateral displacement at the E point decreases by almost 90 percent, while the end moment in the DF member increases by 36 percent, approximately.

A chair, as a furniture, is subjected to considerable lateral displacements related with the rigidity of the connections. Due to lateral displacements, a possible failure can occur probably at the connection points. Because of that, to determine moments at member ends becomes key, at that point. Consequently, it can be concluded that slope-deflection method is very efficient in furniture analysis, a very convenient meth-od to perform the structural analysis, especially of such a frame structure under lateral loading. Present mathematical formulations can be extended more by consid-ering the second order effects, such like axial and shear deformations.

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